Bloch oscillations in tight-binding lattices with defects

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Defects in tight-binding lattices generally destroy the onset of Bloch oscillations (BOs) and the periodic self-imaging of the wave packet due to the lack of an equally spaced Wannier-Stark ladder spectrum. Here it is shown that localized and extended defects in the lattice can be engineered to be transparent for BOs. Such lattices are synthesized from the defect-free lattice by the technique of intertwining operators generally employed in supersymmetric quantum mechanics. The energy spectrum of the synthesized lattices differs from the Wannier-Stark ladder of the defect-free lattice because of the missing of pairs of resonances in the ladder, thus ensuring the persistence of BOs.

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I. INTRODUCTION

Bloch oscillations (BOs) represent a fundamental coherent transport phenomenon originally predicted for quantum particles in periodic potentials driven by an external dc force. BOs have been observed in a wide variety of quantum and classical physical systems, including semiconductor superlattices,[1] ultracold atoms,[2] Bose-Einstein condensates,[3] arrays of evanescently coupled optical waveguides,[4,5] photonic superlattices,[6] and acoustical superlattices.[7] In the absence of dephasing and scattering processes and for a negligible Zener tunneling, the particle accelerated by the external force undergoes an oscillatory (rather than a translational) motion owing to Bragg scattering off the lattice. The periodicity of the motion is basically ascribed to the transition of the energy spectrum from a continuous band with delocalized Bloch eigenstates in absence of the external force to a discrete ladder spectrum with localized Wannier-Stark (WS) eigenstates when the external force is applied and Zener tunneling is negligible. A single-band tight-binding lattice in one spatial dimension provides the simplest model to describe the onset of BOs and the formation of WS localized states.[8,9] The inclusion of lattice disorder, nonlinearities, inhomogeneities or defects has generally a detrimental effect on BOs. Several authors have investigated the onset of BOs in generalized tight-binding lattice models. Among others, we mention BOs in nonlinear lattices[10] and in aperiodic lattices with long-range correlated disorder,[11] BOs with spatially inhomogeneous dc fields,[12] BOs in quasicrystals,[13] BOs for interacting bosons,[14] BOs in disordered lattices with interparticle interaction,[15] and BOs in lattices with inhomogeneous intersite couplings.[16] The introduction of localized or extend defects in the lattice, for either the intersite couplings or the site energies, is generally expected to destroy the WS ladder spectrum of the defect-free lattice, and thus the onset of BOs. In this work it is shown that localized and extended defects in the lattice can be engineered to be transparent for BOs. Such lattices are synthesized starting from the defect-free lattice Hamiltonian by application of the technique of intertwining operators, successfully employed in supersymmetric quantum mechanics[17,18] to add or delete energy states of a given Hamiltonian. In our case, the spectrum of the synthesized lattices differ from the Wannier-Stark ladder of the defect-free lattice because of the missing of pairs of resonances in the ladder, which ensures the persistence of BOs and self-imaging of a wave packet. The paper is organized as follows. In Sec. II, the technique of intertwining operators for spectral engineering of tight-binding Hamiltonians is presented. In Sec. III, the technique is then applied to synthesize lattice models with defects that support BOs. Finally, Sec. IV outlines the main conclusions.

II. TIGHT-BINDING LATTICE ENGINEERING

Let us consider a one-dimensional tight-binding lattice described by the Hamiltonian

$$H = \sum_n \kappa_n(|n-1\rangle\langle n| + |n\rangle\langle n-1|) + \sum_n V_n|n\rangle\langle n|, \qquad (1)$$

where $|n\rangle$ is a Wannier state localized at site n of the lattice, κ_n is the hopping rate between sites $|n-1\rangle$ and $|n\rangle$, and V_n is the energy of Wannier state $|n\rangle$ in presence of the external applied force. To ensure the Hermiticity of H, the hopping amplitudes κ_n and site energies V_n must assume real values. For a defect-free lattice and a homogenous force, the intersite coupling κ_n is independent of n ($\kappa_n=\kappa$) and the site energy V_n increases linearly with n, i.e., $V_n=-Fn$, where F is the potential gradient. The corresponding tight-binding Hamiltonian will be denoted in the following by H_0 and referred to as the WS Hamiltonian. As is well known, its spectrum is purely discrete and the allowed energy levels are given by $E_l=lF$, where $l=0,\pm 1,\pm 2,\ldots$.[9] Our goal is to synthesize a lattice Hamiltonian, of the form given by Eq. (1), with an energy spectrum which differs from that of the WS Hamiltonian H_0 because of the missing of some resonances in the WS ladder spectrum. This spectral engineering problem can be solved by the technique of intertwining operators generally employed in supersymmetric quantum mechanics,[17] which is also valid for matrix Hamiltonians (see, for instance, Ref. 18 and references therein). The technique is briefly described in a rather general manner in this section.

Let us indicate by H_1 the tight-binding Hamiltonian defined by Eq. (1) with hopping amplitudes and site energies given by $\{\kappa_n^{(1)}, V_n^{(1)}\}$, and let us assume that $\kappa_n \to \kappa$ as $n \to \pm\infty$ and that $|V_n|$ is bounded or diverges with an algebraic law $|V_n| \sim n$ as $n \to \pm\infty$; note that such assumptions are sat-

isfied for the WS Hamiltonian H_0. Let us indicate by $\mu_1,\mu_2,\mu_3,\ldots$ the point spectrum of H_1 and let $|\phi^{(1)}\rangle = \Sigma_n \phi_n^{(1)}|n\rangle$ be the (proper) eigenfunction of H_1 corresponding to the energy μ_1, i.e.,[19]

$$\kappa_n^{(1)}\phi_{n-1}^{(1)} + \kappa_{n+1}^{(1)}\phi_{n+1}^{(1)} + V_n^{(1)}\phi_n^{(1)} = \mu_1\phi_n^{(1)}, \tag{2}$$

with $|\phi_n^{(1)}| \to 0$ for $n \to \pm\infty$. It can be readily shown that, provided that $\phi_n^{(1)}$ does not vanish, the following factorization for H_1 holds

$$H_1 = Q_1 R_1 + \mu_1, \tag{3}$$

where

$$Q_1 = \sum_n (q_n^{(1)}|n\rangle\langle n| + \bar{q}_{n-1}^{(1)}|n-1\rangle\langle n|), \tag{4}$$

$$R_1 = \sum_n (r_n^{(1)}|n\rangle\langle n| + \bar{r}_{n+1}^{(1)}|n+1\rangle\langle n|) \tag{5}$$

and

$$r_n^{(1)} = -\sqrt{\frac{\kappa_n^{(1)}\phi_{n-1}^{(1)}}{\phi_n^{(1)}}}, \tag{6}$$

$$\bar{r}_n^{(1)} = -\frac{\kappa_n^{(1)}}{r_n^{(1)}}, \tag{7}$$

$$q_n^{(1)} = -r_n^{(1)}, \tag{8}$$

$$\bar{q}_n^{(1)} = -\bar{r}_{n+1}^{(1)}. \tag{9}$$

Let us then construct the new Hamiltonian H_2 obtained from H_1 by interchanging the operators R_1 and Q_1, i.e., let us set

$$H_2 = R_1 Q_1 + \mu_1. \tag{10}$$

Using Eqs. (4)–(9), from Eq. (10) it can be readily shown that H_2 describes the Hamiltonian of a tight-binding lattice [i.e., it is of form (1)] with hopping amplitudes and site energies $\{\kappa_n^{(2)}, V_n^{(2)}\}$ given by

$$\kappa_n^{(2)} = \kappa_n^{(1)}\frac{r_{n-1}^{(1)}}{r_n^{(1)}}, \tag{11}$$

$$V_n^{(2)} = V_n^{(1)} + \kappa_{n+1}^{(1)}\frac{\phi_{n+1}^{(1)}}{\phi_n^{(1)}} - \kappa_n^{(1)}\frac{\phi_n^{(1)}}{\phi_{n-1}^{(1)}}. \tag{12}$$

An interesting property of the new Hamiltonian H_2 is that its energy spectrum is the same as that of H_1, apart from the lack of the discrete energy level $E=\mu_1$. In fact, let us indicate by $|\psi_E\rangle = \Sigma_n \psi_n(E)|n\rangle$ a proper (or improper) eigenfunction of H_1 with energy E. Note that, if E belongs to the point spectrum of H_1, $|\psi_n(E)| \to 0$ as $n \to \pm\infty$, whereas if E belongs to the continuous spectrum of H_1, $|\psi_n(E)|$ remains bounded as $n \to \pm\infty$. Let us first assume that $E \neq \mu_1$. Using factorization (3) for H_1, the eigenvalue equation $H_1|\psi_E\rangle = E|\psi_E\rangle$ reads explicitly

$$Q_1 R_1|\psi_E\rangle = (E-\mu_1)|\psi_E\rangle \tag{13}$$

from which it follows that $R_1|\psi_E\rangle \neq 0$ since $E \neq \mu_1$. Applying the operator R_1 to both sides of Eq. (13), one obtains

$$R_1 Q_1|\tilde{\psi}_E\rangle = (E-\mu_1)|\tilde{\psi}_E\rangle, \tag{14}$$

i.e., $H_2|\tilde{\psi}_E\rangle = E|\tilde{\psi}_E\rangle$, where we have set $|\tilde{\psi}_E\rangle = R_1|\psi_E\rangle$ or, explicitly [see Eq. (5)]

$$\tilde{\psi}_n(E) = r_n^{(1)}\psi_n(E) + \bar{r}_n^{(1)}\psi_{n-1}(E). \tag{15}$$

Therefore, $|\tilde{\psi}_E\rangle$ is an eigenfunction of H_2 corresponding to the energy E. Also, from Eqs. (6), (7), and (15) and from the assumed asymptotic behavior of $\kappa_n^{(1)}$ and $V_n^{(1)}$, it follows that $|\tilde{\psi}_E\rangle$ is a proper (improper) eigenfunction of H_2 in the same way as $|\psi_E\rangle$ is a proper (improper) eigenfunction of H_1. In a similar way, one can show that any eigenvalue E of H_2, belonging to the continuous or to the point spectrum (with $E \neq \mu_1$), is also an eigenvalue of H_2. Therefore the continuous and point spectra of H_1 and H_2 do coincide, apart from the $E=\mu_1$ eigenvalue which needs a separate analysis. For $E=\mu_1$, the (proper) eigenfunction of H_1 is by construction $|\phi^{(1)}\rangle$ [see Eq. (2)], which satisfies the condition $R_1|\phi^{(1)}\rangle = 0$. On the other hand, from Eq. (10) it follows that the eigenvalue equation $H_2|\psi\rangle = \mu_1|\psi\rangle$ is satisfied for either $|\psi\rangle = |\psi^{(1)}\rangle$ or $|\psi\rangle = |\psi^{(2)}\rangle$, where $Q_1|\psi^{(1)}\rangle = 0$ and $Q_1|\psi^{(2)}\rangle = |\phi^{(1)}\rangle$. The equation $Q_1|\psi^{(1)}\rangle = 0$ reads explicitly

$$q_n^{(1)}\psi_n^{(1)} + q_n^{(1)}\psi_{n+1}^{(1)} = 0. \tag{16}$$

Using the expressions of $q_n^{(1)}$ and $\bar{q}_n^{(1)}$ given by Eqs. (6)–(9), the difference Eq. (16) for $\psi_n^{(1)}$ can be solved in a closed form, yielding

$$\psi_n^{(1)} = \frac{1}{\sqrt{\kappa_n^{(1)}\phi_n^{(1)}\phi_{n-1}^{(1)}}}. \tag{17}$$

In view of the asymptotic behavior of $\phi_n^{(1)}$ and κ_n as $n \to \pm\infty$, it turns out that $\psi_n^{(1)}$ is unbounded as $n \to \pm\infty$, i.e., it is not an eigenfunction (neither proper not improper) of H_2. Similarly, as $\phi_n^{(1)} \to 0$ as $n \to \pm\infty$, the equation $Q_1|\psi^{(2)}\rangle = |\phi^{(1)}\rangle$ for $|\psi^{(2)}\rangle$ reduces to Eq. (16) in the asymptotic limit $n \to \pm\infty$, and thus also $\psi_n^{(2)}$ is unbounded at $n \to \pm\infty$. Therefore, none of the two linearly independent solutions $|\psi^{(1)}\rangle$ and $|\psi^{(2)}\rangle$ of the second-order difference equation $H_2|\psi\rangle = \mu_1|\psi\rangle$ are bounded, i.e., μ_1 does not belong neither to the point spectrum nor to the continuous spectrum of H_2.

The factorization method can be iterated to construct new Hamiltonians $H_3,H_4,H_5,\ldots$ whose energy spectra differ from that of H_1 owing to the missing of the discrete energy levels $\{\mu_1,\mu_2\},\{\mu_1,\mu_2,\mu_3\},\{\mu_1,\mu_2,\mu_3,\mu_4\},\ldots$ For instance, to construct the Hamiltonian H_3, let $|\psi\rangle$ be the (proper) eigenfunction of H_1 corresponding to the energy $E=\mu_2$, and let us set $|\phi^{(2)}\rangle = R_1|\psi\rangle$. From the previous analysis, it follows that $|\phi^{(2)}\rangle$ is the (proper) eigenfunction of H_2 corresponding to the energy $E=\mu_2$, i.e.,

$$\kappa_n^{(2)}\phi_{n-1}^{(2)} + \kappa_{n+1}^{(2)}\phi_{n+1}^{(2)} + V_n^{(2)}\phi_n^{(2)} = \mu_2\phi_n^{(2)}. \tag{18}$$

Let us then construct the new operators R_2 and Q_2, defined as in Eqs. (4)–(9) but with $\phi_n^{(1)}$ and $\kappa_n^{(1)}$ replaced by $\phi_n^{(2)}$ and $\kappa_n^{(2)}$, respectively. The factorization $H_2 = Q_2 R_2 + \mu_2$ then holds. Reversing the order of the R and Q operators, one obtains the new Hamiltonian $H_3 = R_2 Q_2 + \mu_2$, which possesses the same energy spectrum of H_1, except for the missing of

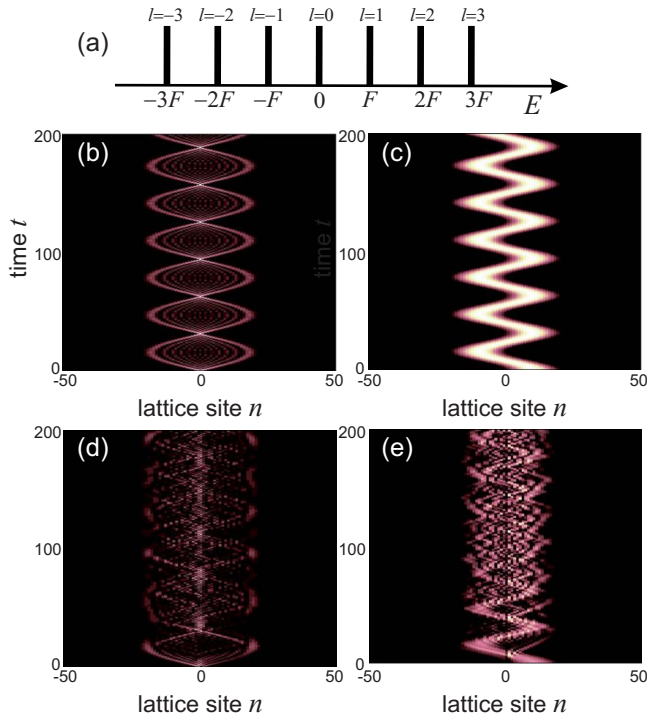

FIG. 1. (Color online) (a) Schematic of the energy levels (WS ladder) of a WS Hamiltonian H_0 ($\kappa_n=\kappa, V_n=-Fn$); the energy level spacing is uniform and equal to F. (b) and (c) Periodic breathing and oscillatory modes in a WS Hamiltonian corresponding to $F=0.2$, $\kappa=1$ and to single-site excitation [$c_n(0)=\delta_{n,0}$ in (b)] and broad Gaussian wave packet excitation [$c_n(0)=\exp[-(n-10)^2/64]$ in (c)] of the lattice at initial time $t=0$. Note the periodic self-imaging of time evolution at multiplies of the BO period $T_B=2\pi/F$. (d) and (e): same as (b) and (c), but for the WS Hamiltonian with a defect in the hopping rate ($\kappa_n=1$ for $n\neq 1$, $\kappa_1=1.5$).

the two energy levels $E=\mu_1$ and $E=\mu_2$. The hopping amplitudes $\kappa_n^{(3)}$ and site energies $V_n^{(3)}$ of the lattice corresponding to the Hamiltonian H_3 are given by Eqs. (11) and (12), with $\phi_n^{(1)}$, $r_n^{(1)}$, $\kappa_n^{(1)}$, and $V_n^{(1)}$ replaced by $\phi_n^{(2)}$, $r_n^{(2)}$, $\kappa_n^{(2)}$, and $V_n^{(2)}$, respectively. It should be noted that the technique of intertwining operators so far described could generate lattice Hamiltonians with complex-valued hopping rates κ_n or site energies V_n; for instance, the hopping amplitudes of H_2 might become complex-valued when $\phi_n^{(1)}$ does not have a defined sign [see Eq. (6) and (11)]. This situation, corresponding to a non-Hermitian lattice with real-valued energy spectrum, will not be considered in this work. It should be nevertheless observed that iteration of the intertwining operator technique could finally restore the Hermiticity of the lattice Hamiltonian, in spite some of the intermediate Hamiltonians are not self-adjoint. This is precisely the case of our interest that will be discussed in the next section.

III. BLOCH OSCILLATIONS

Let us consider the WS Hamiltonian H_0, defined by Eq. (1) with $\kappa_n=\kappa$ and $V_n=-Fn$. As is well known (see, e.g., Ref. 8), H_0 has a purely point spectrum (the WS ladder spectrum) with energies [see Fig. 1(a)]

$$E_l = lF \tag{19}$$

and corresponding localized eigenstates

$$|u^{(l)}\rangle = \sum_n J_{n+l}(\gamma)|n\rangle, \tag{20}$$

where $\gamma=2\kappa/F$ and $l=0,\pm 1,\pm 2,\ldots$ is the quantum number.[9] Note that the WS state with quantum number l is localized at around the lattice site $n=-l$. Owing to the equal spacing of WS modes, the temporal evolution of any wave packet is periodic and self-imaging is attained at times multiples of the Bloch period $T_B=2\pi/F$. This effect is clearly visible by observing the breathing or oscillatory modes[8] corresponding to either an initial single-site or broad-site excitation of the lattice, as shown in Figs. 1(b) and 1(c). The introduction of some defects in the lattice, in either the hopping amplitudes κ_n or site energies V_n, generally breaks the self-imaging property of the lattice [see, as an example, the simulations shown in Figs. 1(d) and 1(e)]. In this section we aim to construct tight-binding lattices with defects in which the self-imaging phenomenon of the WS (defect-free) Hamiltonian H_0 is maintained. This goal can be achieve by the application of the intertwining operator technique described in the previous section assuming as the starting Hamiltonian H_1 the WS Hamiltonian H_0, i.e., $H_1=H_0$. The main idea is that any new Hamiltonian $H_2,H_3,H_4,\ldots$ obtained from H_0 by successive application of the intertwining operator technique has a spectrum which differs from that of H_0 by the missing of some of the WS resonances, and thus a periodic temporal dynamics of the wave packet is maintained with the same period T_B of the original WS Hamiltonian. As a first step, let us construct the Hamiltonian H_2 by removing from the spectrum of the WS Hamiltonian H_0 one WS resonance, for instance, the one with energy $\mu_1=0$ corresponding to the quantum number $l=0$ in Eq. (19). According to Eqs. (11) and (12) with $\kappa_n^{(1)}=\kappa$, $V_n^{(1)}=-Fn$, and $\phi_n^{(1)}=J_n(\gamma)$, the hopping amplitudes and site energies of the lattice associated to H_2 read explicitly

$$\kappa_n^{(2)} = \kappa\sqrt{\frac{J_n(\gamma)J_{n-2}(\gamma)}{J_{n-1}^2(\gamma)}}, \tag{21}$$

$$V_n^{(2)} = -Fn + \kappa\frac{J_{n+1}(\gamma)}{J_n(\gamma)} - \kappa\frac{J_n(\gamma)}{J_{n-1}(\gamma)}. \tag{22}$$

A typical behavior of $[\kappa_n^{(2)}]^2$ and $\Delta V_n^{(2)}\equiv V_n^{(2)}-V_n^{(1)}=V_n^{(2)}+Fn$, as predicted by Eqs. (21) and (22), is shown in Fig. 2. Note that $\kappa_n^{(2)}\to\kappa$ and $\Delta V_n^{(2)}$ settles down to constant values as $n\to\pm\infty$, so that far from the defect near $n=0$ the lattice described by H_2 behaves like a defect-free WS lattice. Note also that, owing to the asymptotic behavior of Bessel functions J_n at large indices, one has $\Delta V_n^{(2)}\to 0$ as $n\to+\infty$ but $\Delta V_n^{(2)}\to 2\kappa/\gamma=F$ as $n\to-\infty$ [see Fig. 2(b)]. Unfortunately, due to the oscillating behavior of Bessel functions $J_n(\gamma)$, the hopping rates $\kappa_n^{(2)}$ can become complex-valued and the Hamiltonian H_2, correspondingly, ceases to be non-Hermitian. This is clearly shown in Fig. 2(a), where $[\kappa_n^{(2)}]^2$ becomes negative at a few lattice sites near $n=0$. This circumstance indicates that, for such indices, the hopping rates

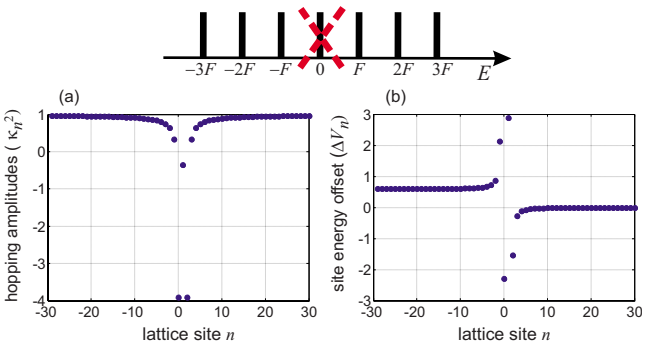

FIG. 2. (Color online) Behavior of (a) the square of hopping amplitudes $[\kappa_n^{(2)}]^2$, and (b) of site energy offset $\Delta V_n^{(2)}=V_n^{(2)}+Fn$ for the Hamiltonian H_2 synthesized from the WS Hamiltonian H_0 by removal of the WS resonance $\mu_1=0$ (see the inset). Parameter values are $\kappa=1$ and $F=0.6$.

are purely imaginary. In spite of the non-Hermiticity of H_2, its spectrum remains real-valued and BOs with the same period T_B as that of the WS Hamiltonian are found.[20] Fortunately, a second application of the intertwining operator technique, assuming as a second energy level μ_2 of H_0 one of the two WS resonances adjacent to μ_1, i.e., $\mu_2=\pm F$ [corresponding to the quantum number $l=\pm 1$ in Eq. (19)], the Hermiticity of H_3 is restored. As an example, Fig. 3 shows the behaviors of $[\kappa_n^{(3)}]^2$ and $\Delta V_n^{(3)}\equiv V_n^{(3)}-V_n^{(1)}$ corresponding to $\mu_2=-F$ and for two values of γ. Note that, for $n\to\pm\infty$, $\kappa_n^{(3)}\to 1$, and $\Delta V_n^{(3)}$ settles down to constant values $[\Delta V_n^{(3)}\to 0$ for $n\to+\infty$, $\Delta V_n^{(3)}\to 2F$ for $n\to-\infty]$ so that the synthesized lattice described by H_3 behaves like the WS (defect-free) lattice for site indices far from $n=0,1$, i.e., far from the lattice sites of WS localized states removed by the intertwining operator technique. As F is decreased, i.e., the localization length of the WS states is increased [see Eq. (20)], the localization length of the defect in the lattice is increased, as one can see by comparing Figs. 3(a) and 3(b) with Figs. 3(c) and 3(d). The persistence of BOs in such synthesized lattices, corresponding to either single-site excitation at $t=0$ or to a multiple-site excitation with a broad Gaussian wave packet, is demonstrated in Figs. 3(e)–3(h). In the figures, the numerically computed evolutions of $|c_n(t)|^2$ versus t, as obtained by solving the Schrödinger equation for the Hamiltonian H_3 with the initial condition $c_n(0)=\delta_{n,0}$ [for the breathing BO modes, Figs. 3(e) and 3(g)] and $c_n(0)=\exp[-(n-10)^2/64]$ [for the oscillatory BO modes, Figs. 3(f) and 3(h)] are depicted for two values of F.

Tight-binding lattices with engineered hopping rates and energy sites corresponding to the ones shown in Fig. 3 could be realized using arrays of evanescently coupled optical waveguides with engineered size and distance, in which the distances between adjacent waveguides control the hopping rates κ_n whereas the channel widths (or refractive index changes) of the guides set the values of the site energies E_n (see, for instance, Refs. 4, 16, and 21 and references therein).

The technique of intertwining operators can be iterated by removing additional resonances from the WS ladder of H_0. Extended numerical simulations show that the resulting Hamiltonians turn out to be Hermitian provided that couples

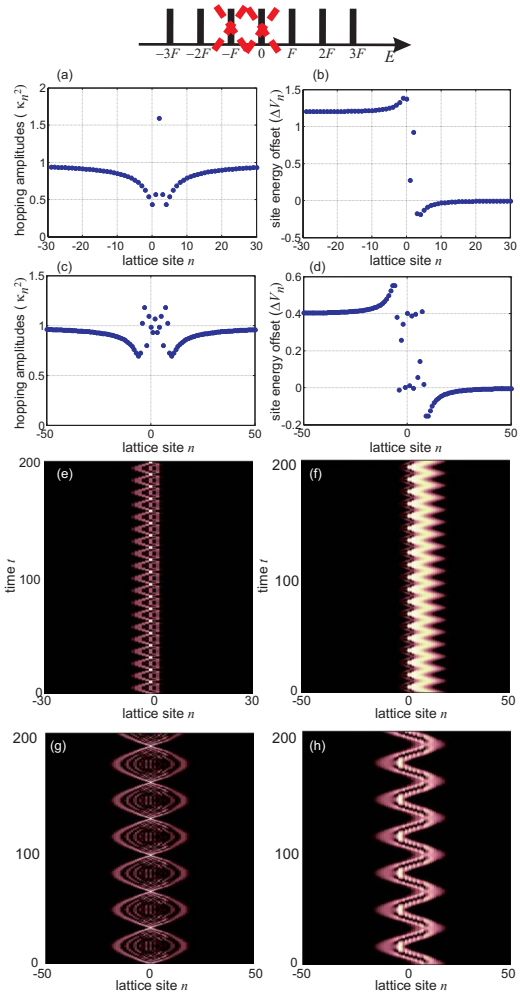

FIG. 3. (Color online) (a–d) Behavior of the square of hopping amplitudes $[\kappa_n^{(3)}]^2$ and of site energy offset $\Delta V_n^{(3)}=V_n^{(3)}+Fn$ for the Hamiltonian H_3 synthesized from the WS Hamiltonian H_0 by removal of the WS resonances $\mu_1=0$ and $\mu_2=-F$ (see the inset at the top of the figure). Parameter values are $\kappa=1$ and $F=0.6$ in (a) and (b), and $\kappa=1$ and $F=0.2$ in (c) and (d). (e) and (f): persistent BOs (breathing and oscillatory modes) in the synthesized lattice corresponding to $F=0.6$. (g) and (h): Same as (e) and (f), but for the synthesized lattice with $F=0.2$.

of adjacent WS resonances are removed, whereas in the other cases the hopping rates can become imaginary at some lattice sites. As a general rule, the removal of a new WS resonance couple at energies $E=lF,(l\pm 1)F$ introduces in the lattice new defects localized at around the lattice site $n=l$. For the Hamiltonian H_{2s+1} obtained by removing from H_0 s couples of adjacent WS resonances, one has $\kappa_n^{(2s+1)}\to 1$ for $n\to\pm\infty$, $\Delta V_n^{(2s+1)}\to 0$ for $n\to+\infty$, and $\Delta V_n^{(2s+1)}\to 2sF$ for $n\to-\infty$. As an example, Fig. 4 shows the behavior of hopping amplitudes and site energy offsets for a synthesized lattice obtained by removal of the four WS resonances with energies $E=0,-F,-25F,-26F$. As the number of removed WS resonances (and hence of defects in the lattice) increases, the behaviors of hopping amplitudes κ_n and site energies V_n become highly irregular, as shown in the example of Fig. 5. Here the Hamiltonian H_{13} is synthesized by successive application of the intertwining operator technique that removes

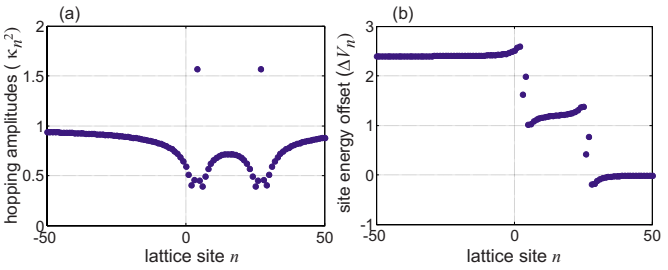

FIG. 4. (Color online) Same as Fig. 2, but for the Hamiltonian H_5 synthesized from H_0 by removing the four WS resonances $E=0,-F,-25F,-26F$. Parameter values are $\kappa=1$ and $F=0.6$.

from the WS ladder spectrum the 12 resonances $E=42F,41F,26F,25F,11F,10F,0,-F,-11F,-12F,-30F,-31F$. It is remarkable that, in such a rather irregular lattice with extended defects, BOs still persists and exact self-imaging is attained, as shown in Figs. 5(c) and 5(d).

As a final comment, it should be noted that the hopping rates and site energies of the synthesized lattices described by the Hermitian Hamiltonians $H_3,H_5,H_7,\ldots$ depend on γ, i.e., on the amplitude of the forcing F entering in H_0. Therefore, a change of the forcing parameter F in the original WS Hamiltonian H_0 gives *different* lattice realizations for $H_3,H_5,H_7,\ldots$ (see, for instance, Fig. 3). This means that, as an additional term $H_p=-fn|n\rangle$ to H_0 simply changes the BO period (just because the external forcing is changed from F to $F+f$), the addition of $H_p=-fn|n\rangle$ to $H_3,H_5,H_7,\ldots$ destroys the onset of BOs. This is shown in Fig. 6(a), where the temporal evolution of the site occupation probabilities $|c_n(t)|^2$ is shown for the Hamiltonian H_{13} of Fig. 5, with an added perturbation term $H_p=-fn|n\rangle$ for $F=0.2$ and $f=-0.1$. For comparison, the evolution of site occupation probabilities for

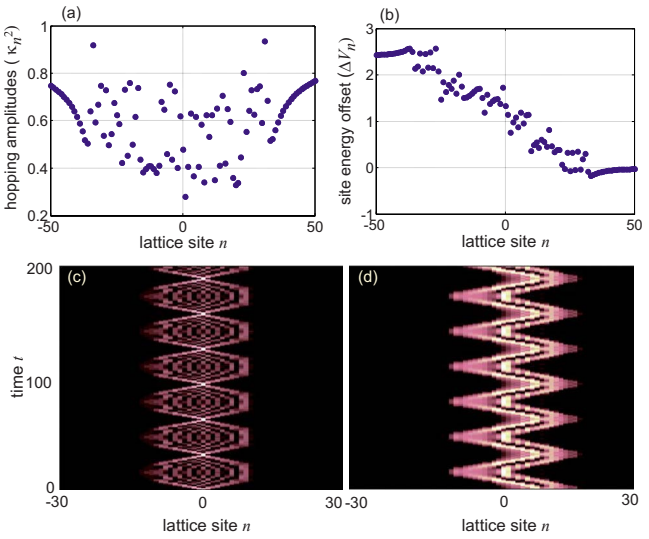

FIG. 5. (Color online) (a) and (b) Same as Fig. 4, but for the Hamiltonian H_{13} synthesized from H_0 by removing twelve WS resonances (parameter values are $\kappa=1$ and $F=0.2$). In (c) and (d), the persistence of breathing and oscillatory BO modes, corresponding to either single-site excitation $c_n(0)=\delta_{n,0}$ [(c)] and to a broad Gaussian wave packet excitation $c_n(0)=\exp[-(n-10)^2/64]$ [(d)], are demonstrated.

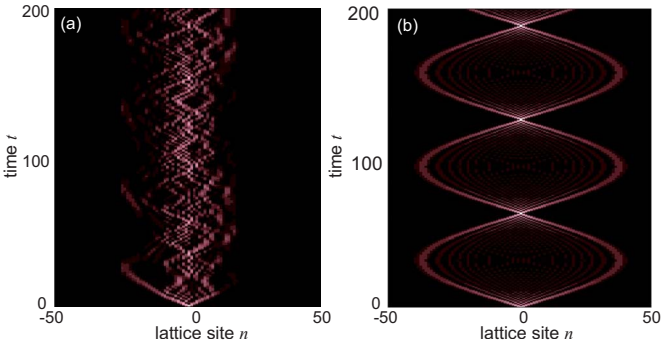

FIG. 6. (Color online) (a) Destruction of BOs (breathing modes) for the lattice Hamiltonian H_{13} of Fig. 5 due to the addition of the perturbation $H_p=-fn|n\rangle$. Parameter values are $\kappa=1$, $F=0.2$, and $f=-0.1$. In (b) the BOs with varied period, corresponding to the addition of H_p to the WS lattice Hamiltonian H_0, are shown for comparison.

the lattice with the WS Hamiltonian H_0 perturbed with $H_p=-fn|n\rangle$ (for the same values of F and f) is depicted in Fig. 6(b). Therefore, while in a WS ladder Hamiltonian BOs are observed for any value of forcing F (a change of F corresponds to a change of the BO period), in the synthesized lattices $H_3,H_5,H_7,\ldots$ a change of the forcing is detrimental for the onset of BOs. Yet, it is remarkable that at a fixed forcing strength BOs can be observed in tight-binding lattices with defects, and even in greatly irregular lattices (like the one shown in Fig. 5).

IV. CONCLUSIONS

In conclusion, in this work it has been shown that single and multiple defects in a tight-binding Wannier-Stark lattice can be introduced such that BOs and the self-imaging property of the WS lattice are not destroyed. Such lattices are synthesized from the defect-free lattice by the technique of intertwining operators generally employed in supersymmetric quantum mechanics to engineer the spectrum of Hermitian Hamiltonians. The energy spectrum of the synthesized lattices differs from the Wannier-Stark ladder of the defect-free lattice because of the missing of pairs of resonances in the ladder, thus ensuring the persistence of BOs. It is envisaged that the lattice engineering technique proposed in this work could be extended to other coherent dynamical regimes, such as dynamic localization in presence of an ac (time-periodic) force.[22] It is also envisaged that the possibility to realize non-Hermitian tight-binding lattices with real-valued energies-mentioned in this work-would deserve further investigation and could stimulate studies in the framework of the rapidly developing field of non-Hermitian quantum mechanics.[20,23]

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